O K L A H O M A S T A T E U N I V E R S I T Y

S C H O O L O F E L E C T R I C A L A N D C O M P U T E R E N G I N E E R I N G

ECEN 5713 Linear System
Fall 1998
Final Exam

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Problem 1:
Find the *observable* canonical form realization (in minimal order) from SISO continuous-time system given below:

$$5t^2\ddot{y}(t) + (t-1)\dot{y}(t) + e^{-2t}y(t) = 2\ddot{u}(t) + 2t\dot{u}(t) - t^2u(t)\ .$$

Notice that gain blocks may be *time* dependent. Show the state space representation and its simulation diagram.

Problem 2:

Show that two "discrete-time" state-space representations $\{A, B, C, D\}$ and $\{\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}\}$ are zero-state equivalent if and only if $CA^k B = \tilde{C}\tilde{A}^k \tilde{B}$, $k = 0,1,2,\ldots$, and $D = \tilde{D}$.

Problem 3:

Compute $\exp(At)$ for

$$A = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

Problem 4:

Find an equivalent discrete-time Jordan canonical form dynamical equation of

$$x(k+1) = \begin{bmatrix} 0 & 4 & 3 \\ 0 & 20 & 16 \\ 0 & -25 & -20 \end{bmatrix} x(k) + \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} u(k) \quad .$$

$$y(k) = \begin{bmatrix} -1 & 3 & 0 \end{bmatrix} x(k) + 4u(k)$$

Problem 5:

Consider the equivalent dynamical equations

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

and

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u$$

$$y = \bar{C}\bar{x}$$

where $\bar{x} = Px$. Their adjoint equations are, respectively,

$$\dot{z} = -A^* z + C^* u$$

$$y = B^* z \qquad (1)$$

and

$$\dot{\bar{z}} = -\bar{A}^* \bar{z} + \bar{C}^* u$$

$$y = \bar{B}^* \bar{z} \qquad (2)$$

where A^* and $\bar{A}^*$ are the complex conjugate transposes of A and $\bar{A}$, respectively. Show that Equations (1) and (2) are equivalent and they are related by $\bar{z} = \left(P^{-1}\right)^* z$.